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Cosmological Bounds on the Neutrino Mass (Cont'd):

Next we consider the bounds on the mass of a neutrino weighing heavier than 1 MeV.

(b) $1\ \text{MeV} \ll m_\nu \ll m_{Z,W^\pm}$: such a neutrino becomes non-relativistic before decoupling from the plasma (i.e. freeze out of weak interactions).

Starting at a temperature $T \gg m_\nu$, the neutrino goes through the following stages:

1- $T \gg m_\nu$: ν is relativistic and in thermal equilibrium. Thus:

$$\boxed{n_\nu \sim T^3 \quad , \quad E_\nu \sim T}$$

2- $T \simeq m_\nu$: transistion to the non-relativistic regime. As ν becomes non-relativistic, its equilibrium number density is given by:

$$n_\nu^{eq} = \left(\frac{m_\nu T}{2\pi}\right)^{3/2} \exp\left(\frac{-m_\nu}{T}\right)$$

For $T < m_\nu$, the exponential suppression implies a small n_ν. The Hubble expansion results in a redshift $n_\nu \propto a^{-3} (\propto T^3)$. This, however, is not enough to keep n_ν at its equilibrium value. In fact, n_ν must increase faster than this to track the equilibrium value.

The relevant physical process is annihilation of a neutrino-antineutrino pair to a pair of electron and positron:

$\nu + \bar{\nu} \rightarrow e^- + e^+$

Note that at $T > m_\nu$ this process goes on in both directions. However, when $T < m_\nu$ the inverse process cannot proceed efficiently due to kinematics (average thermal energy of $e^\pm$

will not be enough).

3- $T_{f.o.} \lesssim T < m_\nu$: ν is non-relativistic and in therma equilibrium so long as neutrino-antineutrino annihilation is efficient;

$$\boxed{n_\nu = \left(\frac{m_\nu T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\nu}{T}\right)}$$

The annihilation rate Γ_{ann} is given by:

$$\Gamma_{ann} = n_\nu \langle \sigma_{ann} v_{rel} \rangle$$

Where;

$$\langle \sigma_{ann} v_{rel} \rangle = G_F \, m_\nu^2 \, (\underset{\text{S-wave}}{a} + \underset{\text{P-wave}}{b \langle v^2 \rangle} + \dots) \qquad (*)$$

$G_F \, m_\nu^2$ comes because of weak interaction that govern the annihilation. The term inside the parantheses is a partia wave expansion, which is good in the non-relativistic regime. The first term dominates if $a \neq 0$. But there are cases where $a = 0$, and hence the cross section is suppressed. For the time being, we consider cases where $a \neq 0$, and hence:

$$\langle \sigma_{ann} v_{rel} \rangle \sim G_F^2 m_\nu^2 \Rightarrow \Gamma_{ann} \sim \left(\frac{m_\nu T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\nu}{T}\right) G_F^2 m_\nu^2$$

The expansion rate follows:

$$H = \left(\frac{\pi^2}{90} g_*\right)^{1/2} \frac{T^2}{M_P}$$

Starting at $T \simeq m_\nu$, it is clear that $\Gamma_{ann} \gg H$ initially. However, Γ_{ann} decreases faster than H as T drops (because of the exponential suppression). Thus there exists a temperature $T_{f.o.}$ (called freeze out temperature) at which $\Gamma_{ann} \simeq H$. From then on, annihilation will not be efficient, and n_ν will be subject only to expansion.

To find $T_{f.o.}$ we put $\Gamma_{ann} \simeq H$:

$$\left(\frac{m_\nu T_{f.o.}}{2\pi}\right)^{3/2} \exp\left(-\frac{m_\nu}{T_{f.o.}}\right) G_F^2 m_\nu^2 \simeq \left(\frac{\pi^2}{90} g_*\right)^{1/2} \frac{T_{f.o.}^2}{M_P} \Rightarrow$$

$$\exp\left(\frac{m_\nu}{T_{f.o.}}\right) \simeq (2\pi)^{-3/2}\left(\frac{\pi^2}{90} g_*\right)^{-1/2}\left(\frac{m_\nu}{T_{f.o.}}\right)^{1/2}\left(\frac{m_\nu}{1\ \text{GeV}}\right)^3 G_F^2 M_P$$

$$\Rightarrow \frac{m_\nu}{T_{f.o.}} \simeq -2.76 - \frac{1}{2}\ln\left(\frac{\pi^2}{90} g_*\right) + \frac{1}{2}\ln\left(\frac{m_\nu}{T_{f.o.}}\right) + 3\ln\left(\frac{m_\nu}{1\,\text{GeV}}\right)$$

$$\Rightarrow \boxed{\frac{m_\nu}{T_{f.o.}} \simeq +16.73 - \frac{1}{2}\ln\left(\frac{\pi^2}{90} g_*\right) + \frac{1}{2}\ln\left(\frac{m_\nu}{T_{f.o.}}\right) + 3\ln\left(\frac{m_\nu}{1\,\text{GeV}}\right)}$$

The logarithmic terms do not contribute very much

As a result, we find a robust expression:

$$\boxed{\frac{m_\nu}{T_{f.o.}} \sim 20 + (\text{logarithmic corrections})}$$

Therefore $\nu + \bar{\nu} \to e^+ + e^-$ ceases to be effective

at $T_{f.o.} \sim \frac{m_\nu}{20}$.

4 - $T < T_{f.o.}$: n_ν is subject to expansion only,

and hence $n_\nu \propto a^{-3}$. Since the entropy density $s \propto a^{-3}$

as well, hence $\frac{n_\nu}{s}$ is a constant for $T < T_{f.o.}$.

Note that $\frac{n_\nu}{s}$ will have the same value that it has at the freeze oot:

$$\frac{n_\nu}{s} = \frac{n_\nu^{f.o.}}{s_{f.o.}}$$

$n_\nu^{f.o.}$ can be found using the fact that $\Gamma_{ann} \simeq H$

then:

$$n_\nu^{f.o.} G_F^2 m_\nu^2 \simeq \left(\frac{\pi^2}{90} g_*\right)^{\frac{1}{2}} \frac{T_{f.o.}^2}{M_P} \Rightarrow n_\nu \sim G_F^{-2} m_\nu^{-2} \left(\frac{\pi^2}{90} g_*\right)^{\frac{1}{2}} \frac{T_{f.o.}^2}{M_P}$$

While:

$$s_{f.o.} = \frac{2\pi^2}{45} g_* T_{f.o.}^3$$

Thus:

$$\frac{n_\nu^{f.o.}}{s_{f.o.}} \sim G_F^{-2} m_\nu^{-2} \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \frac{1}{4 T_{f.o.} M_P}$$

And (at $T < T_{f.o.}$):

$$\frac{n_\nu}{s} \sim 5 G_F^{-2} \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \frac{1}{m_\nu^3 M_P}$$

From BBN we know that $\frac{n_B}{\;} \sim 9 \times 10^{-11}$. This results in:

$$\frac{n_N}{n_B} \sim \frac{5}{9} \times 10^{11} G_F^{-2} \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \frac{1}{m_N^3 M_P} \sim 170 \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \left(\frac{1\,\text{GeV}}{m_N}\right)^{3}$$

The contribution of ν to the energy density of the universe cannot exceed that from the dark matter (which is ~ 6 times of baryons). This leads to:

$$n_N\, m_N \leq 6 n_B \times (1\,\text{GeV}) \Rightarrow 170 \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \left(\frac{1\,\text{GeV}}{m_N}\right)^2 \leq 6$$

$$\Rightarrow \left(\frac{m_N}{1\,\text{GeV}}\right)^2 \geq 30 \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{2}} \Rightarrow$$

$$\boxed{\left(\frac{m_N}{1\,\text{GeV}}\right) \geq 5 \left(\frac{\pi^2}{90} g_*\right)^{-\frac{1}{4}}}$$

This is the so-called Lee-Weinberg bound. For $g_* = 10.75$: we find $\underline{m_N \gtrsim 5\,\text{GeV}}$.

To be precise, this result has been obtained when $a \neq 0$ in $(*)$. This is correct for Dirac neutrinos (when neutrino and antineutrino are distinguishable)

In case of indistinguishable neutrinos and antineutrino (called Majorana neutrino) $a = 0$, and hence σ_{ann} is suppressed. The lower bound on the neutrino mass in this case is found to be $m_N \gtrsim 15\,\text{GeV}$.

We note that neutrinos with a mass $m_N \gg 1\,\text{MeV}$ are non-relativistic when they decouple from the plasma (when weak interactions freeze out). They are "cold dark matter" candidates in this case, which is favored by observational data. Hence, unlike the case when $m_N \ll 1\,\text{MeV}$ (hot dark matter), they can account for all of the dark matter in the universe.